Math 3305 Chapter 1 Section 1.2 Script

Now we'll look at a modern "Common Notion" : Logic. This system underlies all the work behind proving Theorems in any geometry. And we don't make a list of them any more at all. We start with "Undefined Terms". But you do need to know a bit of logic to do proofs.

We start with a "statement". A statement is a simple "subject verb object" sentence. No connectives or clauses. No "and", "or", "because" or "since", for example…sentences with words like that are called "compound statements".

For the logic we are doing each statement needs to be verifiably true or false. It should not surprise you to learn that there are OTHER logic systems out there now. BUT we will stick to the traditional one.

Let's look at a statement: "Today it is raining". Suppose it's true. Then "Today it is not raining" is false. We can show both of these in a truth table box. Let's call the true statement "T".

The negation of T is "not T" and it is false. $\sim T$

T $\qquad \sim T = F$

T	T
$\sim T$	F

Popper 1.2, Question 1

The following is a statement: "If Tony is 5 and a boy, ~~the~~ then Toni is also 5 and is a girl."

A. True

B. False

Now usually when we have two statements we call them P and Q. We will join P and Q with connectives to make "compound statements". Let's look at a truth table framework for two statements. It's important to include all of the cases so our table gets quite a bit larger.

P	Q	P and Q
T	T	
T	F	
F	T	
F	F	

Now we'll want to look at connectives, ways to combine statements into basic compound statements. We will use "and", "or", and an implication ("if, then") to do this

And	$\wedge$	$P \wedge Q$	$P \wedge Q$
Or	$\vee$	$P \vee Q$	$P \vee Q$
If, then	$\rightarrow$	$P \rightarrow Q$	$P \rightarrow Q$

Popper 1.2, Question Two

$P \wedge Q$ is read

A. P and Q

B. P or Q

C. If P, then Q

Now let's look at whole truth tables for each of these

And $\wedge$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Or $\vee$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication $\longrightarrow$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

An example of implication from Euclidean Geometry from our book

If two triangles $\triangle$ ABC and $\triangle$ DEF satisfy angle A is congruent to angle D, side AB is congruent to side DE and angle D is congruent to angle E, then the two triangles are congruent. (Thm. 2.1.1 ASA Thm. p. 39)

Popper 1.2, Question Three

Given P true and Q false,

$P \to Q$ is

A. true

B. false

C. neither

D. both

A larger compound statement "if and only if"…is a mathy shorthand. Let's look at this a little more closely

Symbols $\leftrightarrow$ iff

Written out the long way in symbols:

$P \to Q \wedge Q \to P$

short way $P \leftrightarrow Q$

if P then Q AND if Q then P

And an example from Euclidean Geometry from our book

Two sides of a triangle are congruent if and only if the angles opposite these sides are congruent. (Thm 2.1.4, Isosceles Triangle Thm. p. 41)

There are then TWO proofs to do:

If two sides are congruent, then two angles are congruent AND

If two angles are congruent, then two sides are congruent.

Call "two sides are congruent" S and "two angles are congruent" A

The mathiest way to say this is $S \leftrightarrow A$. How concise!

Now let's revisit implication with blank or dummy statements P and Q. There are 4 ways to rearrange these statements using P, Q and negation. Our original implication will be $P \rightarrow Q$. Original statement truth table

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The converse of our original statement is $Q \rightarrow P$.

Keeping our original truth values let's look at this one.

Q	P	$Q \rightarrow P$
T	T	T
F	T	T
T	F	F
F	F	T

Now let's join both of these with "and".

$P \rightarrow Q$	$Q \rightarrow P$	and $\wedge$
T	T	T
F	T	F
T	F	F
T	T	F

$P \leftrightarrow Q$
T
F
F
T

w/ std T. values

Now another logic notion: logically equivalent. If two statements are logically equivalent, they have the same truth table given the same truth value structure.

Given P → Q, the contrapositive is ~Q→ ~P. Let's look at both truth tables.

P → Q

P	Q	→
T	T	T
T	F	F
F	T	T
F	F	T

~Q → ~P

~Q	~P	→
F	F	T
T	F	F
F	T	T
T	T	T

same

Logically equivalent (p. 6, text) Look hard at the example mid-page 6 for this.

Law of the Contrapositive

A sentence of the form P→ Q is true IFF the contrapositive ~Q→ ~P is true. ☆

(box page 8, text) important

Moving right along with the variation on the implication theme, there's the inverse: ~P→ ~Q

~P	~Q	→
F	F	T
F	T	T
T	F	F
T	T	T

Contrapos ~Q → ~P

ck original & contrapos. above

different

So now we're looking at an original implication ($P \rightarrow Q$). And there's the converse ($Q \rightarrow P$), the inverse ($\sim P \rightarrow \sim Q$) and the contrapositive ($\sim Q \rightarrow \sim P$). These can be a little tough to keep track of…and they are all different!

Essay 1.2 One

Work out a mnemonic device to remember which implication goes with which name. OPQ "original pretty question" might be one. Use your imagination and go wild here! But be sure you can remember it.

Wrapping up with a bit more vocabulary:

Popper 1.2, Question Four

Given our original compound statement is $A \rightarrow B$, then $\sim B \rightarrow \sim A$ is

A. called the contrapositive

B. is logically equivalent

C. is an implication in its own right

D. all of the above

E. none of the above

Now finishing up:

Conjecture as opposed to Theorem. If someone calls a sentence or two a Theorem, it's true. If someone labels a sentence or two a Conjecture, it is an open question whether it's true or false.

So on a test, instructions to "prove this theorem" means dig right into proving it. While instructions to "prove the conjecture or find a counterexample" means you need to do some preliminary work to figure out if it is true or not. "Finding a counterexample" means finding ONE time you meet the "if statement considerations" and find that the "then part" is false.

For example here's a conjecture:

If you add a positive number and a negative number, you get a positive number…prove or find a counterexample.

$5 + -8 = -3$

Contrapos!

Stop - full stop

Start by trying some number combinations:

Then stop right there and circle the counterexample. You're done.

For us on a test, usually logic problems come in what I call "one liner" format. Here's one:

For P false and Q true, find the truth value of the statement

$$(\sim P \vee Q) \rightarrow Q$$

P F Q T

~P T

( )

T v T = T

T → T

( ) Q

T

Here's another:

For P true and Q true, find the truth value of the statement

$(P \rightarrow \sim Q) \wedge (Q \rightarrow \sim P)$

$(T \Rightarrow F) \wedge (T \rightarrow F)$

$F \wedge F$

F

Popper 1.2, Question Five

Given P false and Q true, find the truth value of the statement

$(P \rightarrow Q) \vee Q$

A. True

B. False

Popper 1.2, Question Six

$(P \rightarrow Q)$ is true. What is the truth value of

$(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$? Hint: Check back up in contrapositive! p.6 in notes

A. True

B. False

Wrapping up Chapter 1 Section 2 we have

One essay on page 7 in these notes. Turn in under assignments.

Popper 1.2 turn in under EMCFs. 6 questions, 5 answer choices available on all.

And 1.2 problems due with the Chapter 1 Homework (all together with all Chapter 1 problems, turn in under assignments. Do three problems from this section: #1, #4, and #6. Plus turn in your answer to the following question

1.2 Ms. Leigh's Problem One

SMSG Axiom 14 says:

If two angles form a linear pair, then they are supplementary.

What is the contrapositive to this axiom and what is its truth value?

See the course calendar for the due dates!